Exercise 2: Determine whether Sn is an unbiased estimator of σ. In case it is not an unbiased estimator, which one is larger E[Sn] or σ?

Solution:

Consider the sample variance:

= 1/n \* [here, Mn = Sample Mean]

Now,

E [ = 1/n \* E [– 2 \* Mn \* + n \* Mn ^ 2]

= E [(1/n) \*

= E [(1/n) \*

= µ ^ 2 + σ ^ 2 – (µ ^ 2 +

[E(X) ^ 2 = µ ^ 2 + σ ^ 2 and E [Mn ^ 2] = µ ^ 2 + ]

= σ ^ 2

Conclude that is a biased estimator of the variance. Nevertheless, note that if *n* is relatively large, the bias is very small [1]. Since *E* [] = σ ^ 2, so obtain an unbiased estimator of *σ ^ 2* by multiplying by . Thus, define

2 = \*

= \* (

By the above discussion, 2 is an unbiased estimator of the variance. Note that if n is large, the difference between 2 and is very small. Also define the sample standard deviation as

=

Although the sample standard deviation is usually used as an estimator for the standard deviation, it is a biased estimator. To see this, note that *S* is random, so *Var(S)* > 0. Thus,

0 < *Var(S)* = E [ − (E [)2 = σ2 − (E [)2.

Therefore, E [< σ, which means that *S* is a biased estimator of *σ*.

Reference:

1. https://www.probabilitycourse.com/chapter8/8\_2\_2\_point\_estimators\_for\_mean\_and\_var.php